Add and Subtract Unit Fractions

Assessment Type	Recommended Grade Level	MYP Criterion Level
Mathematical Investigation	Grade 6 (MYP1)	MYP 1

MYP Assessment Criteria
Criterion B: Investigating patterns
Criterion C: Communicating

MYP Command Terms Used
Apply, write down, state, analyze, suggest, use, find, verify, show, select, describe, investigate

MYP Global Context
Identities and relationships

MYP Key Concepts
Relationships

MYP Related Concepts
Equivalence, Simplification

MYP Branch of Mathematics
Numerical and abstract reasoning

MYP Topics and Skills

- Number operations
- Operations with fractions with different denominators (addition, subtraction)
- Recognizing simple patterns

Prior Knowledge Needed

- Use the relationships between the operations of addition, subtraction in order to process information to solve problems
- Use fractions to represent whole-part relationships
- Understanding of the difference between numerator and denominator

Assessment Description

In this assessment, students will explore patterns in relation to the addition and subtraction of unit fractions. In Part A, students write down their understanding of unit fractions before considering a pattern for the addition of unit fractions. Then, in Part B, students carry out an unguided investigation in order to find patterns when a unit fraction is subtracted from another unit fraction.

Materials Needed

Pencil, scrap paper, ruler (optional)

Task-specific instructions / Recommendations

None.

Assessment Criterion B: *Investigating patterns*

	Achievement Level Descriptor (MYP1)	Task Specific Descriptor
0	The student **does not** reach a standard described by any of the descriptors below.	
1-2	The student is able to: i. **apply**, **with teacher support**, mathematical problem-solving techniques to recognize **simple patterns** ii. **state** predictions consistent with simple patterns iii. *(not demonstrated at this level)*.	The student is able to: i. **apply** problem solving techniques to confirm the given values (Q1) ii. **write down** the pattern (Q2a) and **state** the definition (Q2b) iii. *(not demonstrated at this level).*
3-4	The student is able to: i. **apply** mathematical problem-solving techniques to recognize **patterns** ii. **suggest** how these patterns work iii. *(not demonstrated at this level)*.	The student is able to: i. **write down** the denominators in the table (Q3) ii. **analyze** the relevant columns and **suggest** a general rule (Q4), (Q5) iii. *(not demonstrated at this level).*
5-6	The student is able to: i. **apply** mathematical problem-solving techniques to recognize **patterns** ii. **suggest relationships** or general rules consistent with findings iii. verify whether patterns work for **another example**.	The student is able to: i. **use** their finding to **find** the missing values (Q6) ii. **suggest**, in details, a rule (Q7) iii. **verify** that their rule works by **showing** the additions (Q8), (Q9)
7-8	The student is able to: i. **select** and **apply** mathematical problem-solving techniques to recognize **correct patterns** ii. **describe patterns as relationships** or general rules consistent with **correct findings** iii. verify whether patterns work for **other examples**.	The student is able to: i. **select** and **apply** a problem-solving technique to carry out the investigation in Part B ii. **describe** their rule in Part B iii. **verify** that their rule works by testing it with another example in Part B

Assessment Criterion C: *Communicating*

	Achievement Level Descriptor (MYP1)	Task Specific Descriptor
0	The student **does not** reach a standard described by any of the descriptors below.	
1-2	The student is able to: i. use **limited** mathematical language ii. use **limited forms** of mathematical representation to present information iii. communicate through lines of reasoning that are **difficult to understand** iv. *(not demonstrated at this level)*.	The student is able to: i. **use** limited mathematical language in both parts of the investigation ii. **use** limited forms of mathematical representation to present information in both parts of the investigation iii. **explain** through lines of reasoning that are difficult to understand in both parts of the investigation iv. *(not demonstrated at this level).*
3-4	The student is able to: i. use **some appropriate** mathematical language ii. use **appropriate forms** of mathematical representation to present information **adequately** iii. communicate through lines of reasoning that are **able to be understood**, although these are **not always coherent** iv. **adequately organize** information using a logical structure.	The student is able to: i. **use** some appropriate mathematical language in both parts of the investigation ii. **use** appropriate forms of mathematical representation to present information adequately in both parts of the investigation iii. **explain** through lines of reasoning that are able to be understood although not always coherent in both parts of the investigation iv. adequately **organize** information using a logical structure in both parts of the investigation
5-6	The student is able to: i. **usually** use **appropriate** mathematical language ii. **usually** use **appropriate forms** of mathematical representation to present information **correctly** iii. communicate through lines of reasoning that are **usually coherent** iv. **present** work that is **usually organized** using a logical structure.	The student is able to: i. **use** some appropriate mathematical language in both parts of the investigation ii. **use** appropriate forms of mathematical representation to present information correctly in both parts of the investigation iii. **explain** through lines of reasoning that are usually coherent in both parts of the investigation iv. **organize** information using a somewhat logical structure in both parts of the investigation
7-8	The student is able to: i. **consistently** use **appropriate** mathematical language ii. **consistently** use **appropriate forms** of mathematical representation to present information **correctly** iii. communicate **clearly** through **coherent** lines of reasoning iv. present work that is **consistently organized** using a logical structure.	The student is able to: i. **use** appropriate mathematical language consistently in both parts of the investigation ii. consistently **use** appropriate forms of mathematical representation to present information correctly in both parts of the investigation iii. **explain** through lines of reasoning that are clear and coherent in both parts of the investigation iv. **organize** information using a consistently logical structure in both parts of the investigation

Part A: Addition of Unit Fractions

While Estella was in math class she started playing around with fractions and realized she could make any unit fraction as the *addition* of two *different* unit fractions. She wanted to see if this was a pattern and started to investigate.

Tasks

(1) **Apply** problem solving techniques to confirm that the two examples below are correct.

[B: 1-2, i]

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$$

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$$

Estella continued with organizing this pattern in the table given below. *(Note: there is no need to complete this table below at this time.)*

Column 1	Column 2	Column 3
Resulting Fraction	**First Fraction**	**Second Fraction**
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$
$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{20}$
$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{30}$
$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{42}$
$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{56}$
$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{72}$
$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{90}$

Table 1

(2)

a. **Write down** a pattern you notice with all the numerators in Table 1.

All the numerators have the value of 1.

b. All the fractions in Table 1 are unit fractions. Based on the pattern you noticed with the numerators, **state** a definition for what you believe a unit fraction is.

[B: 1-2, ii]

A unit fraction is a fraction with a numerator of 1.

Now, let's look at the *denominators* and investigate further.

(3) Use the values given in the first four rows of Table 1 to **write down** the denominators used in Table 2 given below.

Column 1	Column 2	Column 3
Denominator of Resulting Fraction	**Denominator of First Fraction**	**Denominator of Second Fraction**
2	3	6
3	4	12
4	5	20
5	6	30

Table 2

[B: 3-4, i]

(4) **Analyze** Column 1 and Column 2 in Table 2 to **suggest** a general rule that you notice between the values in these two columns.

[B: 3-4, ii]

Possible answers:

- As you move from Column 1 to Column 2 the values increase by 1.
- (The value of Column 1) + 1 = (The Value of Column 2)

(5) **Analyze** Column 3 in Table 2 to **suggest** how the values in Column 1 and Column 2 may be used to find the value in Column 3.

[B: 3-4, ii]

Possible answers:

- By multiplying the values in Column 1 and Column 2, we get the values in Column 3.
- (The value of Column 1) X (The Value of Column 2) = (The Value of Column 3)

(6) **Use** your previous findings to **find** the missing values in the last four rows of Table 1. Then, go back and complete the table with those values.

[B: 5-6, i]

(7) Given the resulting fractions in Column 1 of Table 1, **suggest**, in detail, a rule for finding the other two unit fractions that can be added together to get the resulting fraction.

[B: 5-6, ii]

Starting with the resulting fraction, the first fraction can be found by adding 1 to the denominator of the resulting fraction. Then multiply the denominators of the resulting fraction and of this new fraction to get the denominator of the final unit fraction.

(8) **Verify** your results by showing that the fractions in Column 2 and Column 3 indeed add up to the fraction in Column 3, in each row. **Show** your additions with all relevant steps below.

[B: 5-6, iii]

Show that $\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$:

$$\frac{1}{5} + \frac{1}{20} = \frac{4}{20} + \frac{1}{20} = \frac{5}{20} = \frac{1}{4}$$

Show that $\frac{1}{5} = \frac{1}{6} + \frac{1}{30}$:

$$\frac{1}{6} + \frac{1}{30} = \frac{5}{30} + \frac{1}{30} = \frac{6}{30} = \frac{1}{5}$$

Note: In the next four additions, write down the missing fractions in the empty space below.

Show that $\frac{1}{6} =$ $\quad$ + $\quad$:

$$\frac{1}{7} + \frac{1}{42} = \frac{6}{42} + \frac{1}{42} = \frac{7}{42} = \frac{1}{6}$$

Show that $\frac{1}{8} =$ $\quad$ + $\quad$:

$$\frac{1}{9} + \frac{1}{72} = \frac{8}{72} + \frac{1}{72} = \frac{9}{72} = \frac{1}{8}$$

Show that $\frac{1}{7} =$ $\quad$ + $\quad$:

$$\frac{1}{8} + \frac{1}{56} = \frac{7}{56} + \frac{1}{56} = \frac{8}{56} = \frac{1}{7}$$

Show that $\frac{1}{9} =$ $\quad$ + $\quad$:

$$\frac{1}{10} + \frac{1}{90} = \frac{9}{90} + \frac{1}{90} = \frac{10}{90} = \frac{1}{9}$$

(9) **Use** your method to **show** that the rule suggested in question (7) works by **finding** the two unit fractions that add up to $\frac{1}{12}$.

[B: 5-6, iii]

The denominator of the first fraction is $12 + 1 = 13$.
The denominator of the second fraction is $12 \times 13 = 156$

We have

$$\frac{1}{13} + \frac{1}{156} = \frac{12}{156} + \frac{1}{156} = \frac{13}{156} = \frac{1}{12}$$

Therefore, the rule works.

Part B: Subtraction of Unit Fractions

Josh has been observing Estella playing around with unit fractions and asks the following question:

Are there are patterns to discover for subtracting unit fractions?

Investigate subtracting a unit fraction from another unit fraction, such as the following:

$$\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$$

$$\frac{1}{3}-\frac{1}{4}=\frac{4}{12}-\frac{3}{12}=\frac{1}{12}$$

Read all the instructions before starting this investigation:

- **Selecting** and **apply** problem solving techniques of your own to carry out the investigation.

  [B: 7-8, i]
- **Describe** a rule that links the three fractions used together.

  [B: 7-8, ii]
- **Verify** your rule works by testing it with another example.

  [B: 7-8, iii]
- Make sure to:
  - **Use** appropriate mathematical vocabulary that has been discussed throughout the unit.

    [C: 1-8, i]
  - **Show** this investigation in a **clear**, **coherent**, and **organized** manner, that includes **multiple forms of representation** such as **tables, calculations,** and **supporting explanations**.

    [C: 1-8, ii-iv]

Carry out this investigation on this and the next page. (Note: it is recommended that you do your work on scrap paper before writing it here.)

Column 1	Column 2	Column 3
First Fraction	**Second Fraction**	**Resulting Fraction**
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{12}$
$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{20}$
$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{30}$

Denominator

Column 1	Column 2	Column 3
Denominator of First Fraction	**Denominator of Second Fraction**	**Denominator of Resulting Fraction**
2	3	6
3	4	12
4	5	20
5	6	30

Rules for denominators:

- The second unit fraction can be found by adding one to the denominator of the first unit fraction.
- The resulting unit fraction can be found by multiplying the denominators of the first and second unit fractions.

All the fractions involved in the pattern are unit fractions so they always have a numerator of 1.

To verify let's check that $\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$ from the given rules.

Given $\frac{1}{5}$ as the first fraction the setup is as follows:

$\frac{1}{5}-?=?$

Denominator of 2nd fraction = $\frac{1}{5+1}=\frac{1}{6}$

$\frac{1}{5}-\frac{1}{6}=?$

$\frac{6}{30}-\frac{5}{30}=?, \quad ?=\frac{1}{30}$

$\frac{6}{30}-\frac{5}{30}=\frac{1}{30}$

$\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$

$\frac{1}{5}\times\frac{1}{6}=\frac{1}{30}$

The rule works